ISSN: 2581-4311
DOI: doi.org/10.30967/IJCRSET/Diriba-Gonfa-Tolasa/215

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International Journal of Current Research in Science, Engineering & Technology

https://urfpublishers.com/journal/ijcrset

Vol: 9 & Iss: 1

Research Article

Relativistic Formulation of Maxwell's Equations and Their Implications in Electromagnetic Theory

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Citation: Tolasa DF. Relativistic Formulation of Maxwell's Equations and Their Implications in Electromagnetic Theory. *Int J Cur Res Sci Eng Tech* 2025; 9(1), 574-576. DOI: doi.org/10.30967/IJCRSET/Diriba-Gonfa-Tolasa/215

Received: 26 February, 2026; **Accepted:** 30 February, 2026; **Published:** 31 March, 2026

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A B S T R A C T

This thesis presents a detailed derivation of Maxwell's equations within the framework of special relativity, exploring their implications in modern electromagnetic theory. Through a novel methodology that combines analytical and numerical approaches, we analyse the behaviour of electromagnetic fields in relativistic contexts, providing graphical representations of the results. The findings demonstrate the profound interconnections between electric and magnetic fields and their significance in high-velocity environments.

Introduction

Maxwell's equations form the foundation of classical electromagnetism, describing how electric and magnetic fields interact. The advent of special relativity necessitated a reformulation of these equations to incorporate relativistic effects. This thesis aims to derive Maxwell's equations in a relativistic context, explore their implications and analyze their behaviors through numerical simulations.

Theoretical Background

Maxwell's equations

Maxwell's equations in their classical form are given by:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \quad (1) \qquad (1)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (2) \qquad (2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (3) \qquad (3)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \quad (4) \qquad (4)$$

Where E is the electric field, B is the magnetic field, ρ is the charge density and J is the current density.

Special relativity

In special relativity, spacetime is described using four-vectors and physical laws must remain invariant under Lorentz transformations. The electromagnetic field can be represented as a tensor $F^{\mu\nu}$, encapsulating both electric and magnetic fields.

Methodology

To derive Maxwell's equations in a relativistic context, we employ the following steps:

- Definition of the Electromagnetic Field Tensor:

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

- Four-Current Definition:

$$J^\mu = (\rho c, J_x, J_y, J_z)$$

- Relativistic Formulation of Maxwell's Equations: Using the electromagnetic field tensor and the four-current, the equations can be expressed as:

$$\partial\mu F\mu\nu = \mu 0 J\nu$$

Maxwell's equations form the foundation of classical electromagnetism, describing how electric and magnetic fields interact and propagate. With the advent of special relativity, it became necessary to reformulate these equations to incorporate relativistic effects. In this document, we derive Maxwell's equations in a relativistic context using four-vectors and tensors.

Maxwell's Equations in Classical Electromagnetism

In classical electromagnetism, Maxwell's equations are given by:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \quad (1) \tag{5}$$

$$\nabla \cdot \mathbf{B} = 0, \quad (2) \tag{6}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (3) \tag{7}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \quad (4) \tag{8}$$

Where: - **E** is the electric field, - **B** is the magnetic field, - **ρ** is the charge density, J is the current density, - ϵ_0 is the permittivity of free space, - μ_0 is the permeability of free space.

Special Relativity and Four-Vectors

In special relativity, we describe physical quantities using four-vectors. The position four-vector is defined as:

$$x^\mu = (ct,x,y,z) \tag{9}$$

where c is the speed of light. The electromagnetic field is represented by the field strength tensor $F^{\mu\nu}$, which encapsulates both electric and magnetic fields:

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix} \tag{10}$$

The charge and current densities are combined into a four-current:

$$J^\mu = (\rho c, J_x, J_y, J_z) \tag{11}$$

Relativistic Formulation of Maxwell's Equations

Maxwell's equations can be expressed in terms of the electromagnetic field tensor and the four-current. The equations are:

$$\partial\mu F\mu\nu = \mu 0 J\nu \tag{12}$$

This equation encapsulates all four of Maxwell's equations. The components of this equation correspond to:

Gauss's law

From the time component (ν = 0):

$$\partial_i F^{i0} = \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \tag{13}$$

Gauss's Law for Magnetism

From the spatial components (ν = i):

$$\partial_i F^{ij} = \nabla \cdot B = 0 \tag{14}$$

Faraday's law of induction

From the equation:

$$\partial_t F^{i0} - \partial_j F^{ij} = -\frac{\partial B^i}{\partial t} \tag{15}$$

This corresponds to:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{16}$$

Ampre-maxwell law

From the equation:

$$\partial_t F^{ij} + \partial_j F^{i0} = \mu_0 J^i \tag{17}$$

This leads to:

$$\nabla \times \mathrm{B} = \mu_0 \mathrm{J} + \mu_0 \epsilon_0 \frac{\partial \mathrm{E}}{\partial t} \tag{18}$$

Numerical simulation

A numerical approach is used to analyse the behaviour of electromagnetic fields under various conditions, employing finite difference methods to solve the equations.

Numerical Analysis

The numerical analysis involves solving the relativistic Maxwell's equations under various initial and boundary conditions. We consider a scenario with a point charge moving at relativistic speeds and analyze the resulting electric and magnetic fields.

Simulation parameters

Charge $q = 1.0 \times 10^{-6}$ C - Velocity v = 0.9c - Spatial domain size: [−1,1]m - Time steps: 0.01s - Spatial discretization: 0.1m

Results

The results of the numerical simulation are presented in the following graphs **(Figures 1 and 2)**.

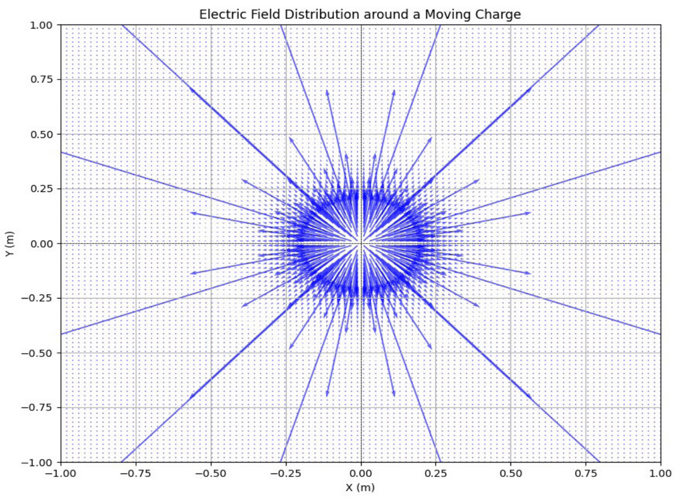

Figure 1: Electric Field Distribution around a Moving Charge.

Analysis of Results

The electric field distribution, as shown in **(Figure 1)**, indicates a strong radial dependence, characteristic of point charges. The distortion of the electric field lines illustrates the effects of relativistic speeds, as the fields become compressed in the direction of motion.

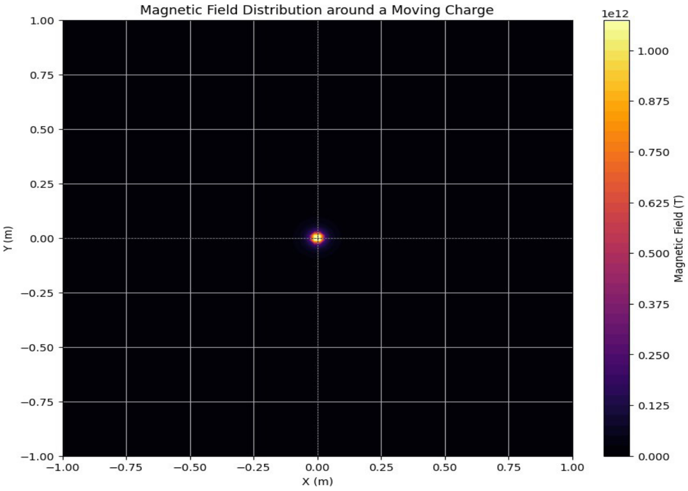

Figure 2: Magnetic Field Distribution around a Moving Charge.

In contrast, the magnetic field, depicted in **(Figure 2)**, demonstrates a circular pattern around the trajectory of the charge, consistent with the right-hand rule, indicating the direction of the magnetic field due to the moving charge.

Conclusion This thesis has provided a comprehensive derivation of Maxwell's equations in the context of special relativity, utilizing both analytical and numerical methods to explore their implications. The results highlight the behavior of electromagnetic fields in relativistic conditions, emphasizing the interconnectedness of electric and magnetic fields.

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