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Analysis and Control of Malaria Dynamic Models

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ABSTRACT

Malaria is one of the infectious and life-threatening vector-borne diseases that causes life-threatening complications. Effective and efficient strategies must be implemented to minimize the damage caused by malaria and to do this, we must understand the dynamics of the measles transmission and implement control methods that are beneficial and cost-effective.

In this work, bifurcation analysis and multiobjective nonlinear model predictive control is performed on two dynamic models involving malaria transmission. Bifurcation analysis is a powerful mathematical tool used to deal with the nonlinear dynamics of any process. Several factors must be considered and multiple objectives must be met simultaneously. The MATLAB program MATCONT was used to perform the bifurcation analysis. The MNLMPC calculations were performed using the optimization language PYOMO in conjunction with the state-of-the-art global optimization solvers IPOPT and BARON.The bifurcation analysis revealed the existence of branch points in both models. The MNLMPC calculations converged to the Utopia solution in both models. The branch points (which cause multiple steady-state solutions from a singular point) are very beneficial because they enable the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in both models.

Keywords: Bifurcation; Optimization; Control; Measles

Introduction

Romero-Leiton, et al.[1] conducted stability analysis and discussed optimal control intervention strategies of a malaria mathematical model. Ibrahim, et al.[2] investigated the impact of awareness on controlling malaria disease using a mathematical modelling approach. Abioye Adesoye Idowu, et al.[3] performed optimal control on a mathematical model of malaria. Kobe[4], developed a mathematical model of controlling the Spread of Malaria Disease Using Intervention Strategies. Ndii, et al.[5] studied the effects of individual awareness and vector controls on malaria transmission dynamics using multiple optimal control. Adepoju, et al.[6] discussed the stability and optimal control of a disease model with vertical transmission and saturated incidence. Keno, et al.[7] discussed optimal control and cost effectiveness analysis of SIRS malaria disease model with temperature variability factor. Aldila Dipo, et al.[8] studied an optimal control problem and backward bifurcation on malaria transmission with vector bias. Tasman, et al.[9] researched an optimal control problem of a malaria model with seasonality effect using real data. Al Basir, et al.[10] explored the effects of awareness and time delay in controlling malaria disease propagation. Sinan Muhammad, et al.[11], developed a Fractional mathematical model of malaria disease with treatment & insecticides. Al Basir, et al.[12] published an article on mathematical modelling and optimal control of malaria using awareness-based interventions. Olaniyi Samson, et al.[13], performed an optimal control analysis of a mathematical model for recurrent malaria dynamics. Wako, et al.[14] analysed and performed optimal control calculations of a model describing malaria and its associated complications.

This work aims to perform bifurcation analysis and multiobjective nonlinear control (MNLMPC) studies in two measles transmission models, which are discussed in Al Basir, et al. (model 1)[13] and Wako, et al, (model 2)[14]. The paper is organized as follows. First, the model equations are presented, followed by a discussion of the numerical techniques involving bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC). The results and discussion are then presented, followed by the conclusions.

Model Equations (Model 1) Al Basir, et al (2023)

In model 1, hu, ha and hi represent the susceptible unaware, susceptible, aware and infected human populations. vs and vi represent the susceptible and infected vector populations, while mv represents the level of awareness. The model equations are

$$\frac{d(hu)}{dt} = \pi h - \alpha hu(mv) - \frac{\lambda 1(hu)vi}{nv} - dh(hu) + \frac{g(ha)}{1+mv}$$
$$\frac{d(ha)}{dt} = \alpha hu(mv) - dh(ha) + c1r(hi)mv - \frac{\lambda 2(ha)vi}{nv} - \frac{g(ha)}{1+mv}$$
$$\frac{d(hi)}{dt} = \frac{\lambda 1(hu)vi}{nv} - (dh + delta)hi - c1(r)(hi)mv + \frac{\lambda 2(ha)vi}{nv}$$
$$\frac{d(vs)}{dt} = \pi v - \frac{\beta(hi)vs}{nv} - \mu vs - c2\gamma(vs)mv$$
$$\frac{d(vi)}{dt} = \frac{\beta(hi)vs}{nv} - \mu vi - c2(\gamma)vi(mv)$$
$$\frac{d(mv)}{dt} = c3\omega + \sigma hi - \theta mv \tag{1}$$

The base parameter values are

$\lambda 1 = 0.02; \alpha = 0.001; \lambda 2 = 0.002; \beta = 0.25; \pi h = 400; \pi v = 10000;$
$\mu = 0.12; r = 0.001; dh = 0.002; \delta = 0.01; \gamma = 0.003; \theta = 0.01;$
$c1 = 0.2; c2 = 0.2; c3 = 0.2; g = 0.01; \ \omega = 0.05; \ \sigma = 0.05;$

c1, c2 and c3 are the control parameters. More details can be found in Al Basir et al (2023).

Model Equations (model 2) Wako, et al (2025)

Model 2 is a scaled model where sh, im, ic, rh, are the scaled variables representing the susceptible humans, malaria-infected humans, those with induced complications and recovered human beings. The scaled susceptible and infected mosquito populations are represented by sv and iv.

The model equations are

$$gfac = \frac{\gamma 1}{1 + \varepsilon ic}$$
$$\frac{d(sh)}{dt} = \mu 1 - \alpha 1 \sigma q(iv)sh - \mu 1 sh$$
$$\frac{d(im)}{dt} = \alpha 1 \sigma q(iv)sh - (\omega 1 + \mu 1)im - \gamma 0(u0)im$$
$$\frac{d(ic)}{dt} = \omega 1(im) - \mu 1(ic) - u1(gfac)ic - \delta 0(ic)$$
$$\frac{d(rh)}{dt} = \gamma 0(u0)im + u1(gfac)ic - \mu 1(rh)$$
$$\frac{d(sv)}{dt} = \mu 2 - \alpha 2\sigma(im)sv - \mu 2(sv)$$
$$\frac{d(iv)}{dt} = \alpha 2(\sigma)im(sv) - \mu 2(iv) \tag{2}$$

The base parameter values are

$\alpha 1 = 0.75; \alpha 2 = 0.2; \sigma = 0.3; \mu 1 = 3.9139e-05; \ \mu 2 = 0.12; \ \omega 1 = 0.2185; \ \gamma 0 = 0.0056;$
$\gamma 1 = 0.0056; \ \delta 0 = 0.0244; \varepsilon = 0.007; q = 2; \ u0 = 0.2; u1 = 0.2;$

More details can be found in Wako et al (2025).

Bifurcation analysis

The MATLAB software MATCONT is used to perform the bifurcation calculations. Bifurcation analysis deals with multiple steady-states and limit cycles. Multiple steady states occur because of the existence of branch and limit points. Hopf bifurcation points cause limit cycles. A commonly used MATLAB program that locates limit points, branch points and Hopf bifurcation points is MATCONT[15,16]. This program detects Limit points (LP), branch points (BP) and Hopf bifurcation points(H) for an ODE system

$$\frac{dx}{dt} = f(x, \alpha) \tag{3}$$

$x \in R^n$ Let the bifurcation parameter be α . Since the gradient is orthogonal to the tangent vector,

The tangent plane at any point $w = [w_1, w_2, w_3, w_4, ...w_{n+1}]$ must satisfy

$$Aw = 0 \tag{4}$$

Where A is

$$A = [\partial f / \partial x \quad | \partial f / \partial \alpha] \tag{5}$$

where $\partial f / \partial x$ is the Jacobian matrix. For both limit and branch points, the Jacobian matrix $J = [\partial f / \partial x]$ must be singular.

For a limit point, there is only one tangent at the point of singularity. At this singular point, there is a single non-zero vector, y, where Jy=0. This vector is of dimension n. Since there is only one tangent the vector $y = (y_1, y_2, y_3, y_4, ...y_n)$ must align with $\hat{w} = (w_1, w_2, w_3, w_4, ...w_n)$. Since

$$J\hat{w} = Aw = 0 \tag{6}$$

the n+1 th component of the tangent vector w_{n+1} = 0 at a limit point (LP).

For a branch point, there must exist two tangents at the singularity. Let the two tangents be z and w. This implies that

$$Az = 0$$
$$Aw = 0 \tag{7}$$

Consider a vector v that is orthogonal to one of the tangents (say w). v can be expressed as a linear combination of z and w ($v = \alpha z + \beta w$). Since $Az = Aw = 0$; $Av = 0$ and since w and v are orthogonal, $w^T v = 0$. Hence $Bv = \begin{bmatrix} A \\ w^T \end{bmatrix} v = 0$ which implies that B is singular.

Hence, for a branch point (BP) the matrix $B = \begin{bmatrix} A \\ w^T \end{bmatrix}$ must be singular.

At a Hopf bifurcation point,

$$\det(2f_x(x,\alpha)@I_n) = 0 \qquad (7)$$

@ indicates the bialternate product while I_n is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in Kuznetsov[17-19].

Multiobjective nonlinear model predictive control (MNLMPC)

The rigorous multiobjective nonlinear model predictive control (MNLMPC) method developed by Flores Tlacuahuaz, et al.[20] was used.

Consider a problem where the variables $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ (j=1, 2..n) have to be optimized simultaneously for a dynamic problem

$$\frac{dx}{dt} = F(x,u) \qquad (8)$$

t_f being the final time value and n the total number of objective variables and u the control parameter. The single objective optimal control problem is solved individually optimizing each of the variables $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ The optimization of $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ will lead to the values q_j^*. Then, the multiobjective optimal control (MOOC) problem that will be solved is

$$\min\left(\sum_{j=1}^{n}\left(\sum_{t_i=0}^{t_i=t_f} q_j(t_i) - q_j^*\right)\right)^2 \qquad (10)$$

$$subject\ to\ \frac{dx}{dt} = F(x,u);$$

This will provide the values of u at various times. The first obtained control value of u is implemented and the rest are discarded. This procedure is repeated until the implemented and the first obtained control values are the same or if the Utopia point where ($\sum_{t_i=0}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j) is obtained.

Pyomo[21] is used for these calculations. Here, the differential equations are converted to a Nonlinear Program (NLP) using the orthogonal collocation method The NLP is solved using IPOPT[22] and confirmed as a global solution with BARON[23].

The steps of the algorithm are as follows

Optimize $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ and obtain q_j^*.

Minimize $\left(\sum_{j=1}^{n}\left(\sum_{t_i=0}^{t_i=t_f} q_j(t_i) - q_j^*\right)\right)^2$ and get the control values at various times.

Implement the first obtained control values

Repeat steps 1 to 3 until there is an insignificant difference between the implemented and the first obtained value of the control variables or if the Utopia point is achieved. The Utopia point is when $\sum_{t_i=0}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j.

Sridhar[24] demonstrated that when the bifurcation analysis revealed the presence of limit and branch points the MNLMPC calculations to converge to the Utopia solution. For this, the singularity condition, caused by the presence of the limit or branch points was imposed on the co-state equation[25]. If the minimization of q_1 lead to the value q_1^* and the minimization of q_2 lead to the value q_2^* The MNLPMC calculations will minimize the function $(q_1 - q_1^*)^2 + (q_2 - q_2^*)^2$. The multiobjective optimal control problem is

$$\min \quad (q_1 - q_1^*)^2 + (q_2 - q_2^*)^2 \quad subject\ to\ \frac{dx}{dt} = F(x,u) \qquad (11)$$

Differentiating the objective function results in

$$\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) = 2(q_1 - q_1^*)\frac{d}{dx_i}(q_1 - q_1^*) + 2(q_2 - q_2^*)\frac{d}{dx_i}(q_2 - q_2^*) \qquad (12)$$

The Utopia point requires that both $(q_1 - q_1^*)$ and $(q_2 - q_2^*)$ are zero. Hence

$$\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) = 0 \qquad (13)$$

The optimal control co-state[25] equation is

$$\frac{d}{dt}(\lambda_i) = -\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) - f_x\lambda_i; \quad \lambda_i(t_f) = 0 \qquad (14)$$

λ_i is the Lagrangian multiplier. t_f is the final time. The first term in this equation is 0 and hence

$$\frac{d}{dt}(\lambda_i) = -f_x\lambda_i; \lambda_i(t_f) = 0 \qquad (15)$$

At a limit or a branch point, for the set of ODE $\frac{dx}{dt} = f(x,u)$ f_x is singular. Hence there are two different vectors-values for $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) > 0$ and $\frac{d}{dt}(\lambda_i) < 0$. In between there is a vector $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) = 0$. This coupled with the boundary condition $\lambda_i(t_f) = 0$ will lead to $[\lambda_i] = 0$ This makes the problem an unconstrained optimization problem and the optimal solution is the Utopia solution.

Results and Discussion

The bifurcation analysis for model 1; with c1 as the bifurcation parameter, revealed a branch point at (hu; ha; hi; vs; vi; mv; c1) values of (175000; 25000; 0; 82918.739635; 0; 1; 3.255054) **(Figure 1a)**.

For the MNLMPC calculations, hu (0) is set to 100000; $\sum_{t_i=0}^{t_i=t_f} ha(t_i)$ were maximized and resulted in a value of 200000 and $\sum_{t_i=0}^{t_i=t_f} hi(t_i)$ was minimized and resulted in a value of 0. The overall optimal control problem will involve the minimization of $(\sum_{t_i=0}^{t_i=t_f} ha(t_i) - 200000)^2 + (\sum_{t_i=0}^{t_i=t_f} hi(t_i) - 0)^2$ was minimized subject to the equations governing the model. This led to a value of zero (the Utopia point).

The MNLMPC values of the control variables, c1, c2 and c3 were 0.5004, 0.5072 and, 0.6403. The various MNMPC figures are shown in **(Figures 1b-1i)**. The control profiles c1, c2, c3 **(Figure 1h)** exhibited noise and this was remedied using the Savitzky-Golay filter to produce the smooth control profiles c1sg, c2sg and c3sg **(Figure 1i)**. It is seen that the presence of a branch point is beneficial because it allows the MNLMPC calculations to attain the Utopia solution, validating the analysis of Sridhar[24].

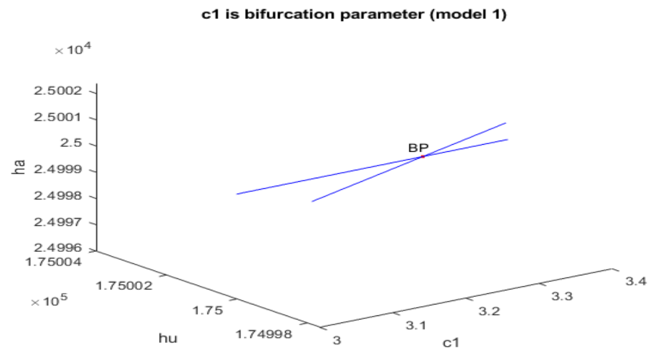

Figure 1a : Bifurcation analysis of model 1.

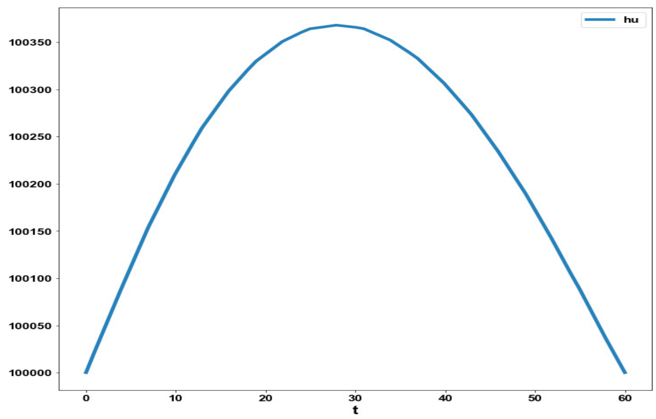

Figure 1b: MNLMPC model 1(hu vs t).

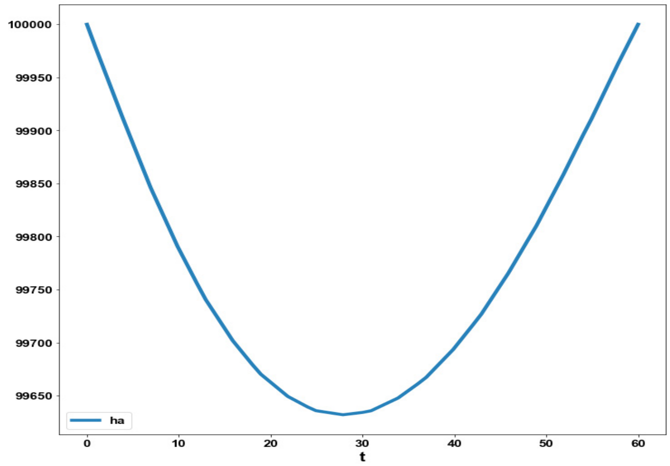

Figure 1c: MNLMPC model 1(ha vs t).

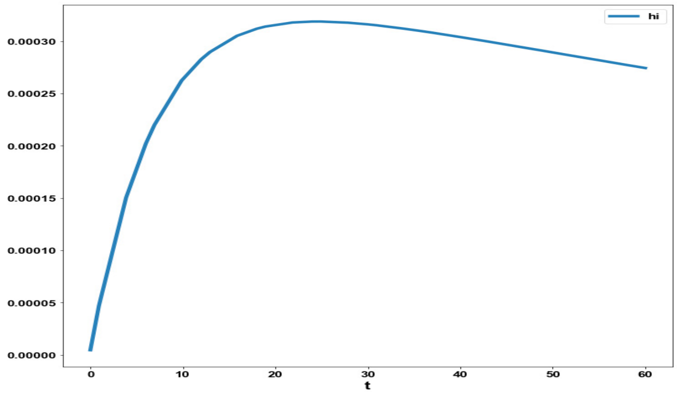

Figure 1d: MNLMPC model 1(hi vs t).

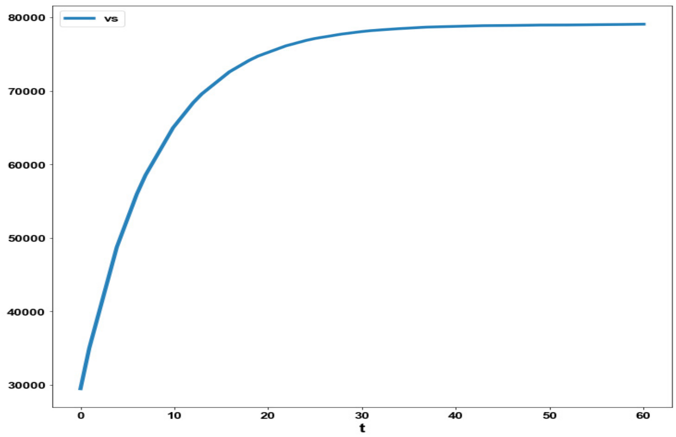

Figure 1e: MNLMPC model 1(vs vs t).

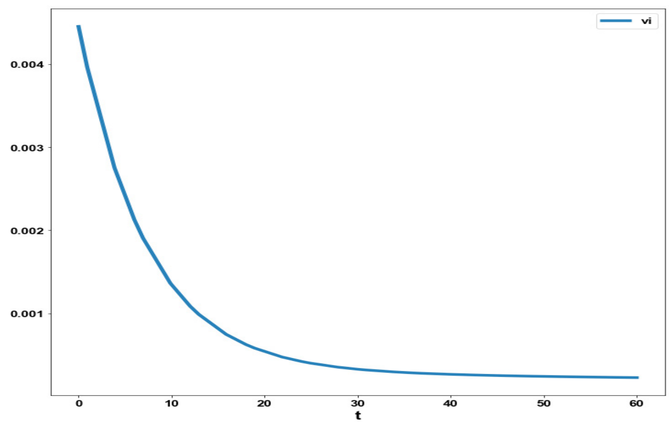

Figure 1f: MNLMPC model 1(vi vs t).

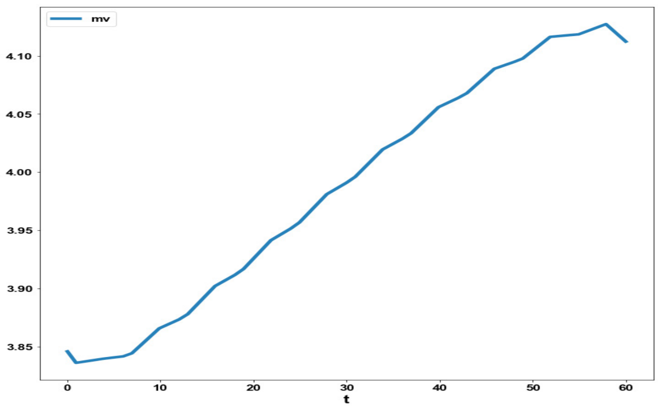

Figure 1g: MNLMPC model 1(mv vs t).

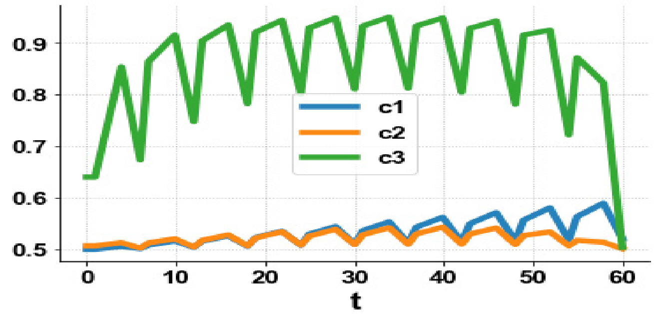

Figure 1h: MNLMPC model 1(u1, u2, u3 vs t).

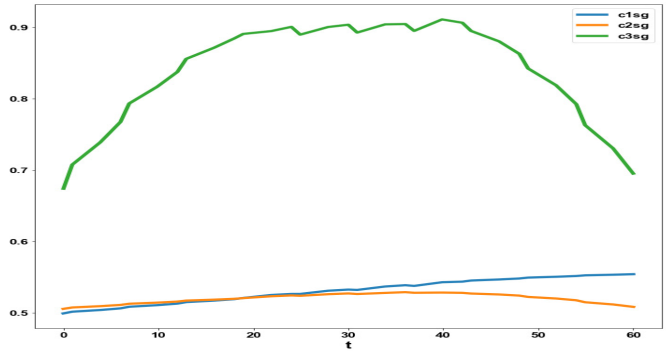

Figure 1i: MNLMPC model 1(u1sg,u2sg,u3sg vs t).

The bifurcation analysis for model 2 with u0 as the bifurcation parameter revealed a branch point at (sh; im; ic; rh; sv; iv; u0) values of (1 0 0 0 1 0 1.153725) **(Figure 2a)**.

For the MNLMPC calculations, sh(0) is set to 0.7; $\sum_{t_i=0}^{t_i=t_f} im(t_i), \sum_{t_i=0}^{t_i=t_f} ic(t_i), \sum_{t_i=0}^{t_i=t_f} iv(t_i)$ were minimized individually and each resulted in a value of 0. The overall optimal control problem will involve the minimization of $(\sum_{t_i=0}^{t_i=t_f} im(t_i)-0)^2+(\sum_{t_i=0}^{t_i=t_f} ic(t_i)-0)^2+(\sum_{t_i=0}^{t_i=t_f} iv(t_i)-0)^2$ was minimized subject to the equations governing the model. This led to a value of zero (the Utopia point).

The MNLMPC values of the control variables, u0, u1 were 0.3778 and 0.3899. The various MNMPC figures are shown in figures 2b-2i. The control profiles u0, u1 **(Figure 2h)** exhibited noise, which was remedied using the Savitzky-Golay filter to produce the smooth control profiles u0sg, u1sg. **(Figure 2i)**. It is seen that the presence of a branch point is beneficial because it allows the MNLMPC calculations to attain the Utopia solution, validating the analysis of Sridhar[24].

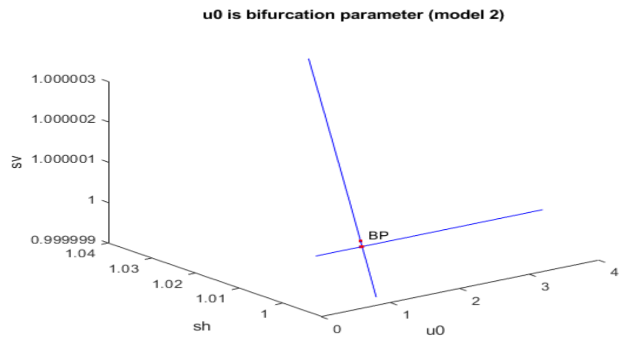

Figure 2a: Bifurcation analysis of model 2.

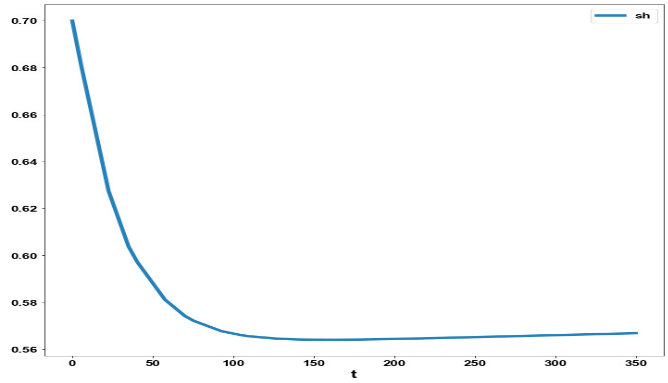

Figure 2b: MNLMPC model 2(sh vs t).

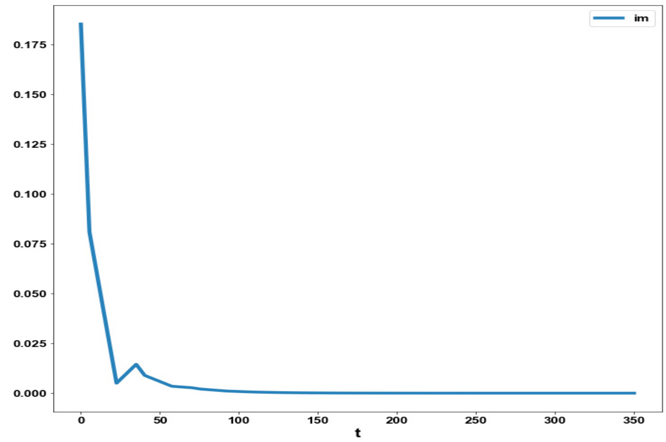

Figure 2c: MNLMPC model 2 (im vs t).

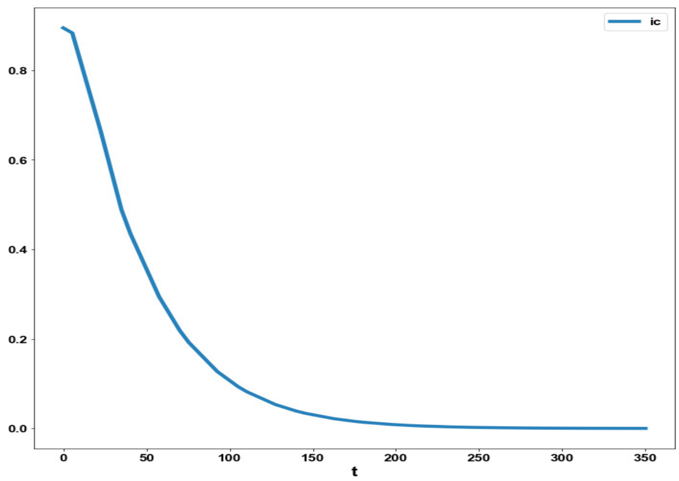

Figure 2d: MNLMPC model 2 (ic vs t).

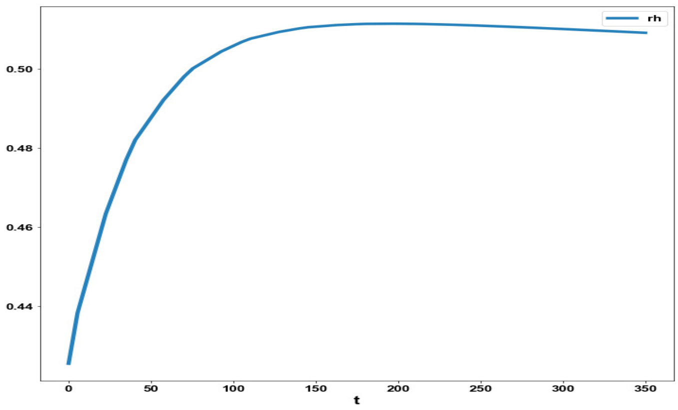

Figure 2e: MNLMPC model 2 (rh vs t).

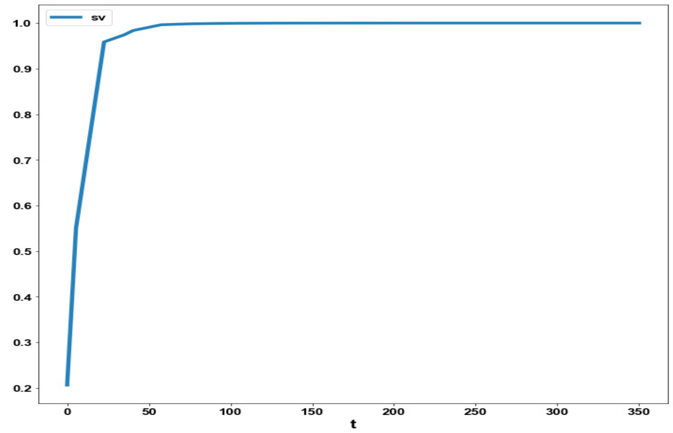

Figure 2f: MNLMPC model 2 (sv vs t).

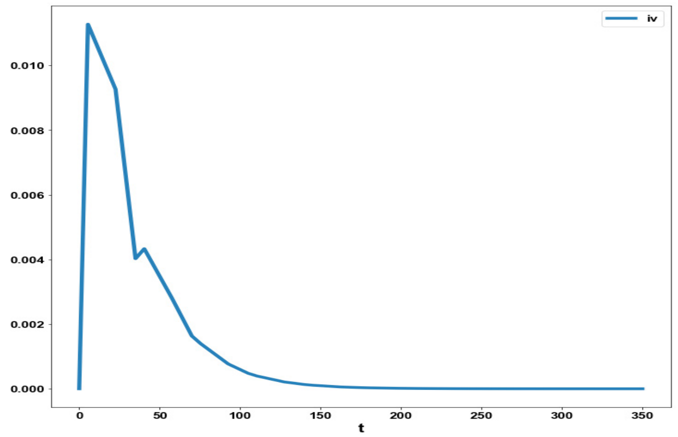

Figure 2g: MNLMPC model 2 (iv vs t).

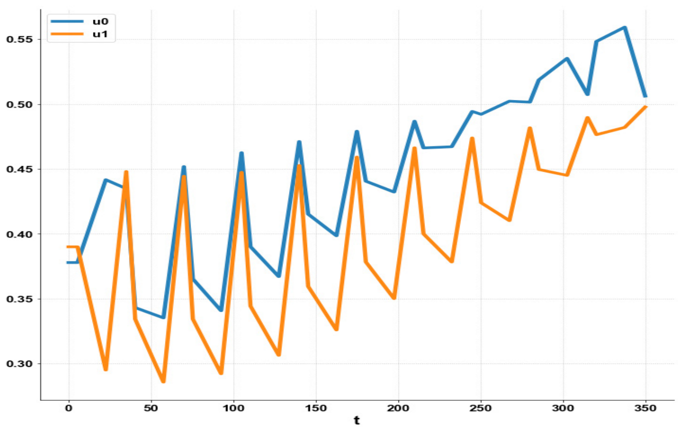

Figure 2h: MNLMPC model 2 (control profiles noise exhibited).

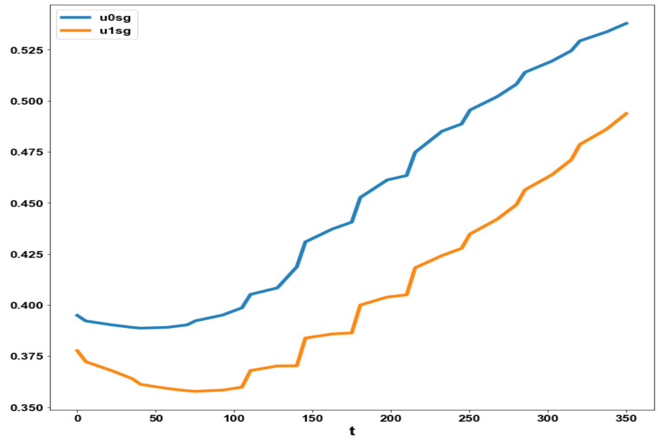

Figure 2i: MNLMPC model 2 (control profiles noise eliminated)

In both models, the presence of a branch point is beneficial because it allows the MNLMPC calculations to attain the Utopia solution, validating the analysis of Sridhar[24].

Conclusions

Bifurcation analysis and multiobjective nonlinear control (MNLMPC) studies in two malaria transmission models. The bifurcation analysis revealed the existence of branch points in both models. The branch points (which cause multiple steady-state solutions from a singular point) are very beneficial because they enable the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in the models. A combination of bifurcation analysis and Multiobjective Nonlinear Model Predictive Control (MNLMPC) for malaria transmission models is the main contribution of this paper.

Data Availability Statement

All data used is presented in the paper.

Conflict of Interest

The author, Dr. Lakshmi N Sridhar has no conflict of interest.

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