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Vol: 1 & Iss: 2

Design of Vibration Suppression Simulation for Flexible Manipulator Model

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Citation: Wang, W., & Sun, Y. (2023). Design of Vibration Suppression Simulation for Flexible Manipulator Model. *J Artif Intell Mach Learn & Data Sci*, 1(2), 65-70.

Received: 10 February, 2023; **Accepted:** 13 June, 2023; **Published:** 30 June, 2023

A B S T R A C T

This paper introduces the domestic and foreign research status quo of the flexible-link manipulator, and to turn the single flexible-link manipulator in the plane as the research object. To establish the dynamic model of flexible-link manipulator system by using the method of modal analysis and Lagrange's equation. This paper also has studied the LQR control theory and has applied it to the flexible-link manipulator system; The simulation model is established in MATLAB Simulink, using system model simulation analysis compared the effect of the vibration control of flexible-link manipulator system. It has pointed out the importance of choosing the appropriate weighting matrix for LQR. By theoretical study and simulation analysis, it has obtained a satisfactory effect of flexible-link manipulator on the vibration suppression.

Keywords: Flexible manipulator, LQR control, Modal analysis, Single flexible arm

Introduction

The establishment of robotics and the birth and development of robots is one of the great achievements of human scientific and technological progress in the 20th century. Robot mechanical arm can be used to replace people for monotonous, repetitive, unsafe mechanical operations; Manipulator is the core subject of robotics research [1].

Research background and significance.

It can be seen from the main literature at home and abroad that the study of flexible arm mainly includes the study of dynamic modeling and the design of flexible arm controller [2]. Dynamic modeling of flexible manipulator is the basis of control, so the quality of dynamic modeling seriously affects the control effect [3].

Dynamic modeling of the flexible arm refers to the establishment of the dynamic equation of the flexible arm. Flexible manipulator system is a complex flexible mechanical system, the quality of the system dynamics model will directly affect the control effect of the system, so modeling is the foundation of flexible manipulator system research, but also the key topic of flexible manipulator system research [4].

The structure of flexible arm includes joint flexibility and link flexibility. Because of the flexibility of its connecting rod, the flexible arm will produce shear deformation, axial deformation and distortion deformation in the process of movement [5]. At present, there are several mainstream methods to describe the deformation of flexible body:

(1) Finite Element method (FEM).

(2) Modal Synthesis (MSM).

(3) Concentrated mass method (LPM).

In the dynamic modeling of flexible arm, the main literature at home and abroad includes the vector mechanics method based on Newton-ruler equation, the dynamics method based on Kane equation and the dynamics method based on Lagrange equation [6].

One of the main objectives of designing and manufacturing flexible arms is to achieve high speed motion and high precision control.

Based on the current literature, it can be roughly divided into the following types of control schemes [7]:

(1) Trajectory tracking control.

(2) PID control.

(3) Nonlinear feedback control.

(4) Adaptive control.

(5) Intelligent control.

Fuzzy Logic Control is an important branch of intelligent Control. Artificial Neural Network Control is also an important branch of intelligent Control. Back Propagation neural network is one of the most widely used neural network models [8].

Future research direction of flexible arm

There are still some problems to be solved:

(1) System modeling of flexible arm: In the study of modeling of more complex flexible arm, the established model simplifies the control requirements as much as possible

(2) System control of flexible arm: in future research, it is more practical to adopt control methods independent of dynamic model (such as fuzzy control).

Dynamic modeling of flexible arm

Dynamic modeling of a single flexible arm.

The flexible arm was regarded as an Euler-Bernoulli beam, and the elastic deformation of the flexible arm was described by modal analysis method. The dynamics equation of the system was deduced according to the Lagrange equation.

(1) Establish coordinate system.

Figure 1 is the mathematical model system of a flexible manipulator with a single link. The manipulator only moves in two dimensions on a plane.

The selected coordinate system is inertial coordinate system. The coordinate system is the moving coordinate system fixedly connected with the flexible arm. The flexible boom is regarded as an Euler-Bernoulli beam, and the shaft and boom coincide without deformation.

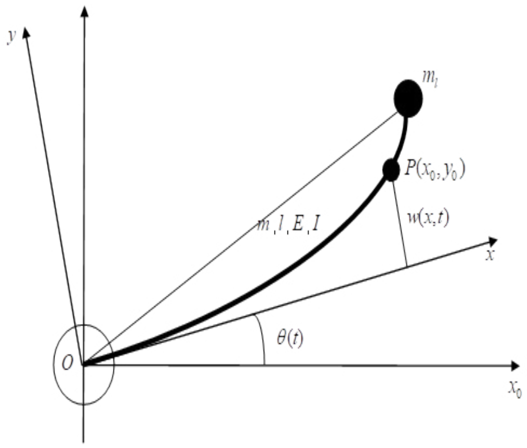

Figure 1: Mathematical model of single link flexible arm

The point is any point on the flexible arm link, and the fixed coordinates are. Expressed in moving coordinates:

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = A(\begin{bmatrix} x \\ y \end{bmatrix} + w(x,t)) \qquad (1)$$

Where, $w(x, t)$ is the lateral elastic deformation of point P on the flexible arm at time t at the point x from O on Ox axis. A is the rotation matrix of coordinate system xOy to inertial coordinate system x_0Oy_0, as follows:

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \qquad (2)$$

Substituting formula (2) into formula (1), the specific expression of point coordinates can be obtained as follows:

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x\cos\theta - w(x,t)\sin\theta \\ x\sin\theta + w(x,t)\cos\theta \end{bmatrix} \qquad (3)$$

Θ is the rigid body rotation angle of the flexible arm at time.

Euler-bernoulli beam lateral free vibration equation. The bending free vibration motion equation of the beam of the flexible arm is:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho S\frac{\partial^2 w(x,t)}{\partial t^2} = 0 \qquad (4)$$

Formula 4 is a fourth-order homogeneous partial differential equation, which can be solved by separating variables.

$$w(x,t) = W(x)q(t) \qquad (5)$$

$W_i(x)$ is the mode function of the flexible arm. $q_i(t)$ is the amplitude of the corresponding mode, which is called modal coordinate.

According to the assumed modal method and vibration analysis theory, the elastic deformation of the flexible arm is:

$$w(x,t) = \sum_{i=1}^{\infty} W_i(x)q_i(t) \qquad (6)$$

is the reserved mode number, abbreviated as the mode number.

In practice, in order to simplify the calculation, only the sum of finite terms is generally taken because the vibration quantity is mainly felt by low order vibration. That is, the elastic deformation formula (7) of the flexible arm can be simplified as:

$$w(x,t) = \sum_{i=1}^{N} W_i(x)q_i(t) \qquad (7)$$

According to Euler-Bernoulli beam theory, formula 4 satisfies the boundary conditions of the flexible arm: the rotation Angle and displacement of the fixed end are equal to 0, namely:

$$\frac{\partial w(0,t)}{\partial x} = 0 \qquad (8)$$

$$w(x,t) = 0 \qquad (9)$$

The bending moment of the beam at shear balance and free end is 0, namely:

$$EI\frac{\partial^3 w(l,t)}{\partial x^3} = m_l\frac{\partial^2 w(l,t)}{\partial x^2} \qquad (10)$$

$$EI\frac{\partial^2 w(l,t)}{\partial x^2} = 0 \qquad (11)$$

Substitute the boundary condition formulas (7), (8), (9), (10) and (11) of the flexible arm into the common formula (4) and take any and. When can be obtained:

$$\rho S\int_0^l W_i(x)W_j(x)dx + m_l W_i(l)W_j(l) = 0 \qquad (12)$$

$$EI\int_0^l \frac{d^2W_i(x)}{dx^2}\frac{d^2W_j(x)}{dx^2} = 0 \qquad (13)$$

ω_i is the natural frequency of order i of the system.

The orthogonality condition of the natural mode of the flexible arm loaded at the end of the equation (12) and (13).

When $i = j$, we can get:

$$\omega_i \left[\rho S \int_0^l sW_i(x)dx + m_l l W_i(l)\right]^2 = EI \int_0^l \left[\frac{d^2W_i(x)}{dx^2}\right] dx \qquad (14)$$

Kinetic energy and potential energy of the system

The kinetic energy of the flexible arm is composed of three parts: kinetic energy T_1 of the driving motor shaft, kinetic energy T_2 of the connecting rod and kinetic energy T_3 of the terminal load. Thus, the total kinetic energy T of the flexible arm is as follows:

$$T = T_1 + T_2 + T_3 \qquad (15)$$

According to the kinetic energy theorem, kinetic energy T_1 of the motor shaft is:

$$T_1 = \frac{1}{2} J_0 \dot{\theta}^2 \qquad (16)$$

Point P is a point on the link of the flexible arm. The derivative of point P with respect to time can be obtained:

$$\begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \end{bmatrix} = \begin{bmatrix} -(x\sin\theta + w(x,t)\cos\theta)\dot{\theta} - \dot{x}\cos\theta - \dot{w}(x,t)\sin\theta \\ (x\cos\theta - y\sin\theta)\dot{\theta} + \dot{x}\sin\theta + \dot{w}(x,t)\cos\theta \end{bmatrix} \qquad (17)$$

Due to $\dot{x} = 0$ in formula (17), the square of the velocity of point P is:

$$v^2 = \dot{x}_0^2 + \dot{y}_0^2 = (x^2 + w^2(x,t))\dot{\theta}^2 + \dot{w}^2(x,t) + 2\dot{\theta}x\dot{w}(x,t) \qquad (18)$$

Therefore, the kinetic energy of the flexible arm connecting rod is:

$$T_2 = \frac{1}{2}\rho S \int_0^l \left[(x^2 + w^2(x,t))\dot{\theta}^2 + \dot{w}^2(x,t) + 2\dot{\theta}x\dot{w}(x,t)\right] dx \qquad (19)$$

The kinetic energy of the load at the end of the flexible arm is:

$$T_3 = \frac{1}{2} m_l \left[(x^2 + w^2(l,t))\dot{\theta}^2 + \dot{w}^2(l,t) + 2\dot{\theta}x\dot{w}(l,t)\right] \qquad (20)$$

Substituting the common formulas (16), (19) and (20) into (15), the total kinetic energy of the system can be obtained:

$$T = T_1 + T_2 + T_3 = \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}\rho S \int_0^l \left[(x^2 + w^2(x,t))\dot{\theta}^2 + \dot{w}^2(x,t) + 2\dot{\theta}x\dot{w}(x,t)\right] dx + \frac{1}{2}m_l\left[(x^2 + w^2(l,t))\dot{\theta}^2 + \dot{w}^2(l,t) + 2\dot{\theta}x\dot{w}(l,t)\right] \qquad (21)$$

Substituting the common formula (7) into the common formula (20), and using the common formula (12) and (13) under the orthogonality condition, the following equation can be obtained:

$$T = \frac{1}{2}(J + \rho S \int_0^l x^2 dx + m_l l^2)\dot{\theta}^2 + \frac{1}{2}\sum_{i=1}^N M_i [\frac{dq_i(t)}{dt}]^2 + \frac{1}{2}\dot{\theta}^2 \sum_{i=1}^N M_i [q_i(t)]^2 + \dot{\theta}\sum_{i=1}^N [\rho S \int_0^l sW_i(x)dx + m_l l W_i(l)]\frac{dq_i(t)}{dt} \qquad (22)$$

The potential energy of elastic deformation of the flexible arm is:

$$V = \frac{1}{2}EI \int_0^l [\frac{\partial^2 w(x,t)}{\partial x^2}]dx \qquad (23)$$

Formula (7) is substituted into formula (23). The potential energy of the flexible arm is:

$$V = \frac{1}{2}EI \int_0^l [\sum_{i=1}^N \frac{d^2W_i(x)}{dx^2} q_i(t)]dx \qquad (24)$$

Dynamic model of flexible arm

According to the orthogonality condition of the natural vibration mode of the suspension beam arm, it can be concluded that:

$$\rho S \int_0^l W_i^2(x)dx + m_l W_i^2(l) = M_i \qquad (25)$$

$$EI \int_0^l [\frac{d^2W_i(x)}{dx^2}]^2 dx = K_i \qquad (26)$$

The dynamic equation of flexible arm with terminal load can be obtained by Lagrange equation:

$$J\ddot{\theta} + \sum_{i=1}^N [\rho S\sigma_i + m_l l W_i(l)]\ddot{q}_i = u(t) \qquad (27)$$

$$[\rho S\sigma_i + m_l l W_i(l)]\ddot{\theta} + M_i\ddot{q}_i + K_i q_i = 0 \qquad (28)$$

The dynamics equation of the flexible arm, equations (27) and (28), are written into matrix formulas. Considering modal damping, the kinetic equation is

$$M\dot{Q} + KQ + C\dot{Q} = Fu \qquad (29)$$

State space description of flexible arm

The state space equation is used to describe the dynamic equation of the flexible arm:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (30)$$

$$y = Cx(t) \qquad (31)$$

It can be obtained from formula (30). Considering the damping effect, the state space equation of the flexible arm is:

$$\dot{x}(t) = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix} u(t) \qquad (32)$$

The numerical simulation

Related parameters of the flexible arm

One end of the flexible arm is connected to a drive motor fixed to the base, and the other end is connected to the end-effector of the load. Related parameters of the flexible arm are shown in Table 1:

Table 1. Classification of objects.

Flexible arm parameter	value
material	45 steel
Material density ρ	$7.8 \times 10^3 kg/m^3$
Elastic modulus E	$2.0 \times 10^{11} N/m^2$
Length l	$1.5m$
Cross-sectional area S	$1.026mm^2$
End load m_l	$0.1kg$
Moment of inertia J_0	$0.8kg \cdot m^2$

According to the geometrical and physical parameters of the flexible arm in Table 1. The natural frequencies of the first three orders of the flexible arm can be obtained by calculation, as shown in Table 2:

Table 2. The first three natural frequencies of the flexible arm.

The first three natural frequencies	$\omega_1 = 1.2518(Hz)$
	$\omega_2 = 7.6173(Hz)$
	$\omega_3 = 39.906(Hz)$

The simulation analysis

Based on the current literature, the first-order elastic vibration mode plays a major role in the vibration variables of the flexible manipulator with single link. The second mode response is very small, and usually only the first mode is taken. Therefore, this paper only takes the first mode.

Take the first mode, set the state variable $x = [\theta, q_1, \dot{\theta}, \dot{q}_1]$, and obtain the state space equation of the flexible arm:

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 19 & 0 & 6 \\ 0 & -105 & 0 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 5 \\ -2 \end{bmatrix} u \qquad (33)$$

Select the end corner of the flexible arm as the output variable, and obtain:

$$C = [1, W_1(l), \cdots W_N(l), 0, 0, \cdots, 0] \qquad (34)$$

Therefore, the output equation of the flexible arm is:

$$y = \dot{\theta} = [1 \quad -0.03 \quad 0 \quad 0]x \qquad (35)$$

The state space description equation of the flexible arm is Equation (33), and the output equation is Equation (35). Formula (33) and Formula (35) are encapsulated in Simulink's state-space to conduct the modeling and simulation of flexible arm Simulink. The end Angle $\dot{\theta}$, moment $u(t)$ and end disturbance $w(l, t)$ of the flexible arm are shown in Figure 2, Figure 3 and Figure 4 respectively:

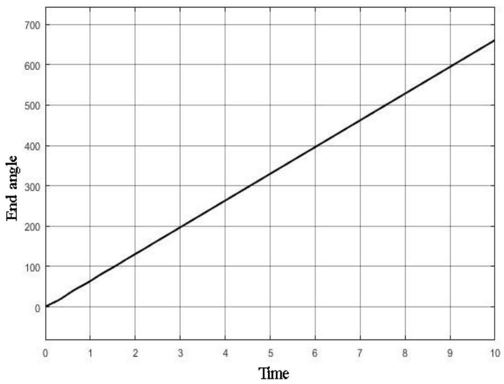

Figure 2: The end Angle of the flexible arm.

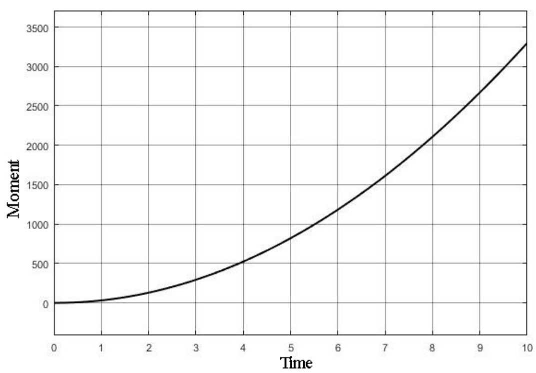

Figure 3: The moment of the flexible arm.

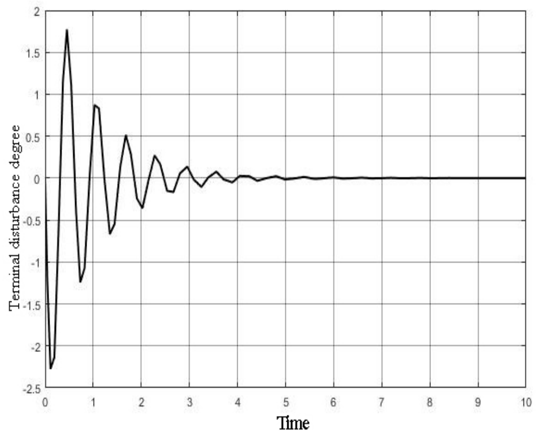

Figure 4: The end disturbance of the flexible arm.

Vibration suppression control and simulation of flexible arm

Introduction to LQR control principle

The Linear Quadratic Regulator is a Linear Quadratic Regulator. Its regulating object is a linear system given in the form of state space equation in modern control theory, and its objective function is a quadratic function controlling input and object state.

Suppose the state space equation and output equation of the system are:

$$\begin{cases} \dot{X} = AX + BU \\ Y = CX \end{cases} \qquad (36)$$

Quadratic performance index function is:

$$J = \frac{1}{2}\int_0^{\infty} x(t)^T Q x(t) + u(t)^T R u(t) dt \qquad (37)$$

LQR optimal control means that when the system deviates from the zero state by external action or interference, it seeks to obtain the minimum input vector U that makes J return to zero state at the same time, and then U is called optimal control. According to the optimal control theory, U meeting the above conditions is:

$$U = -KX = -R^{-1}B^T PX \qquad (38)$$

Where is of the Riccati equation. is the optimal linear feedback gain matrix. The Riccati equation is:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad (39)$$

Then and the optimal feedback gain matrix can be obtained:

$$K = R^{-1}B^T P = [k_1 \quad k_2 \quad k_3 \quad k_4]^T \qquad (40)$$

LQR control of flexible arm

In LQR controller, matrix Q is the weighted matrix of performance indicator function for state quantity, and diagonal matrix is generally selected. The larger the value of the element is, the more important the variable is in the performance function.

The basic steps of the simulation trial and error method are as follows:

(1) The analysis system selected Q and R, and computer simulation.

(2) The optimal gain matrix K is obtained by substituting the required Q and R into the controller.

(3) Substitute K into the controller of the system and run the simulation. If the system performance indicator is not met, select Q and R again until the performance indicator is met.

The simulation analysis

The flexible arm system is a linear time-invariant model. Take the first-order mode, set the state variable $x = [\theta, q_1, \dot{\theta}, \dot{q}_1]$, and the state space model of the flexible arm is:

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 19 & 0 & 6 \\ 0 & -105 & 0 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 5 \\ -2 \end{bmatrix} u \qquad (41)$$

$$y = \dot{\theta} = [1 \quad -0.03 \quad 0 \quad 0]x$$

The quadratic performance index of the controlled system is

$$\dot{x} - \dot{z}_1 = A[x - z_1] + Bu \qquad (42)$$

$$J = \frac{1}{2}\int_0^\infty [(x - z_1)^T C^T Q C (x - z_1) + u^T R u]dt \qquad (43)$$

Balances the performance requirements of system response and control. The value of weighting matrix reflects the importance of system response and control. The values of each element in the matrix and reflect the performance requirements of each control variable and each response in the system.Reference related literature, after several screening, selected

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (44)$$

$$R = 0.1$$

At this point, the state feedback gain can be obtained as :

$$K = [100.0000 \quad -30.9531 \quad 8.8625 \quad 4.5663] \qquad (45)$$

The end Angle Θ of the flexible arm is shown in Figure 5. It can be seen from the figure that after about 5s, the flexible arm reaches the preset corner, and the vibration attenuates quickly when it reaches the position.

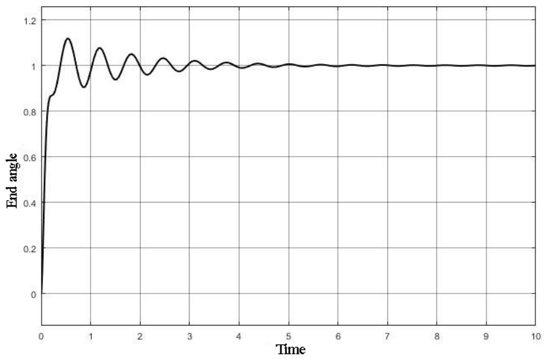

Figure 5: The end Angle of the flexible arm.

The end disturbance $w(l, t)$ of the flexible arm is shown in Figure 6. It can be seen from the figure that the vibration amplitude of the flexible arm is large at the beginning of 2s, and after about 5.5s, the attenuation of the end disturbance of the flexible arm tends to zero.

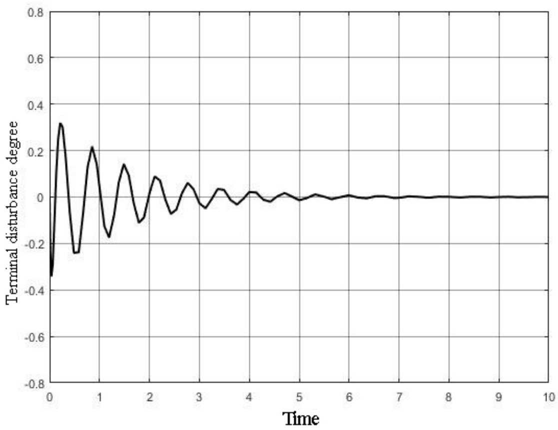

Figure 6: Flexible arm end disturbance.

The control moment $u(t)$ of the flexible arm is shown in Figure 7 (see the next page). According to the figure, the control torque reaches its maximum value at about $t = 0$.

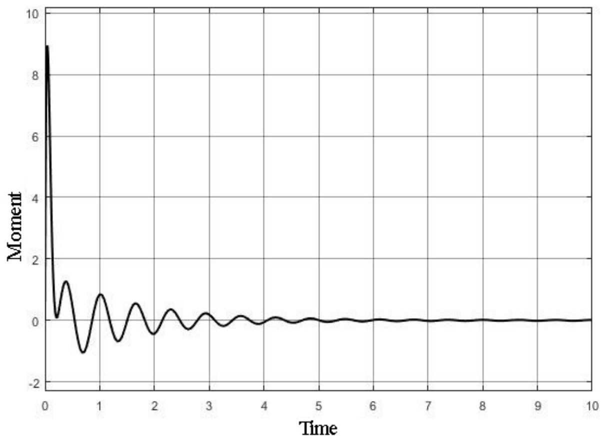

Figure 7: The control torque of the flexible arm.

With reference to relevant literature and after multiple screening, the following items are selected:

$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (46)$$

$$R = 0.5$$

The m program in MATLAB was used to solve the state gain feedback matrix:

$$K = [14.1421 \quad -17.3851 \quad 3.1808 \quad 0.8861] \qquad (47)$$

The end Angle of the flexible arm is shown in Figure 8. It can be seen from the figure that after about 3s, the flexible arm reaches the preset Angle, and the vibration attenuates rapidly when it reaches it.

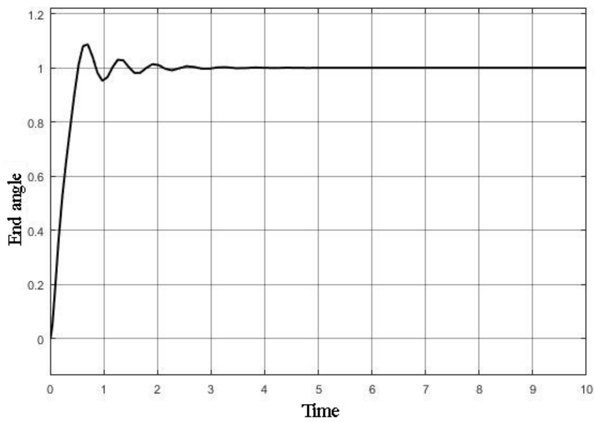

Figure 8: End Angle of flexible arm.

The end disturbance of the flexible arm is shown in Figure 9. According to the figure, the vibration amplitude of the flexible arm is large in the first 1s, and after about 3s, the end disturbance attenuation of the flexible arm tends to zero. Compared with Figure 6, not only the attenuation time is shortened by more than 2s, but also the vibration amplitude is greatly improved. The maximum amplitude is only about 0.13, which is about half of figure 5.

Figure 10 shows the control torque of the flexible arm . According to the figure, the control torque reaches its maximum value at about , and at about 3s, when the flexible arm reaches the preset corner, the control torque also decays to zero. Compared with Figure 7, the instantaneous peak value of control moment also decreases a lot.

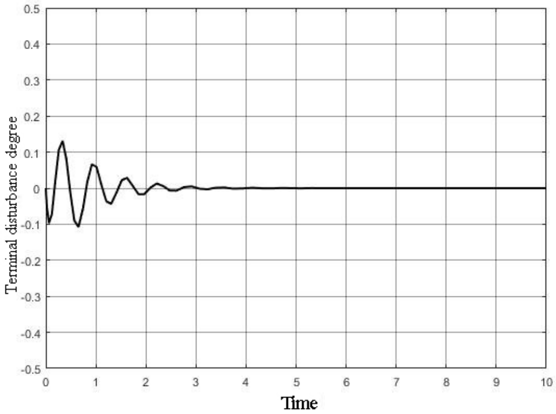

Figure 9: The end disturbance of the flexible arm is.

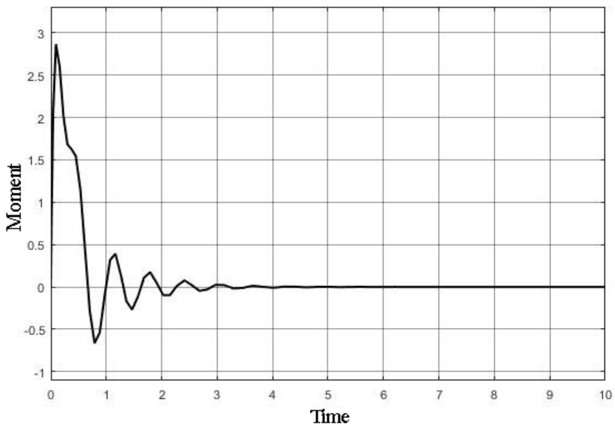

Figure 10: The control torque of the flexible arm.

Conclusion

The modeling and vibration suppression of a single flexible arm are studied in this paper. Based on the Lagrange equation, the dynamics model of a planar flexible manipulator with single link was established. LQR optimal control was added to the flexible manipulator system to suppress the vibration of the flexible manipulator. Simulation experiments were carried out in MATLAB/Simulink.

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